We describe a general method of transforming arbitrary programming languages into probabilistic programming languages with straightforward MCMC inference engines. Random choices in the program are “named” with information about their position in an execution trace; these names are used in conjunction with a database holding values of random variables to implement MCMC inference in the space of execution traces. We encode naming information using lightweight source-to-source compilers. Our method enables us to reuse existing infrastructure (compilers, profilers, etc.) with minimal additional code, implying fast models with low development overhead. We illustrate the technique on two languages, one functional and one imperative: Bher, a compiled version of the Church language which eliminates interpretive overhead of the original MIT-Church implementation, and Stochastic Matlab, a new open-source language.

1 INTRODUCTION

Probabilistic programming languages simplify the development of probabilistic models by allowing programmers to specify a stochastic process using syntax that resembles modern programming languages. These languages allow programmers to freely mix deterministic and stochastic elements, resulting in tremendous modeling flexibility. The resulting programs define prior distributions: running the (unconditional) program forward many times results in a distribution over execution traces, with each trace generating a sample of data from the prior. The goal of inference in such programs is to reason about the posterior distribution over execution traces conditioned on a particular program output. Examples include BLOG [7], PRISM [15], Bayesian Logic Programs [5], Stochastic Logic Programs [9], Markov Logic [14], Independent Choice Logic [12], IBAL [11], Probabilistic Scheme [13], A [10], and HANSEI [6].

We present a general technique for turning any programming language into a probabilistic programming language with an accompanying universal Markov chain Monte Carlo inference engine. Our method allows the full use of all language constructs permitted by the original (non-probabilistic) language by leveraging native compilers (or interpreters). The key technical idea is to give each random choice of a fixed program a unique “name” that depends on its position in a given execution trace. We then convert stochastic functions into deterministic ones which use these names to look up their return values in a database that stores the values of all random choices. By controlling the values in this database, execution traces of the program can be controlled, which allows us to construct the key operations needed for the Metropolis-Hastings algorithm.

Different techniques for naming random choices result in different MCMC dynamics. Our central contribution is to name random choices based on their structural position in the execution trace, which allows fine-grained sharing of random choice values between subsequent MCMC states. We describe how naming information can be provided by a side computation that is constructed by a source-to-source transformation at compile time: the original probabilistic program can be augmented with naming information to create a new program that makes the names available. The resulting new program can be executed at full speed, with only minimal overhead to track names and control the database, and no changes to the compiler or interpreter for the underlying language.

We illustrate our technique on two languages, one functional and one imperative. The first is named “Bher,”
and is a compiler for the Church probabilistic program-
ing language [4]. The current MIT-Church implemen-
tation uses a custom interpreter to reason about the dis-
tribution of execution traces, but since our method does
not incur interpretive overhead, the resulting programs
can run orders of magnitude faster. We also illustrate
the technique on an imperative language by creating
Stochastic Matlab, an entirely new language.

2 Overview of the Method

We begin by outlining our setup. We define an uncondi-
tioned probabilistic program to be a parameterless func-
tion \( f \) with an arbitrary mix of stochastic and deter-
nistic elements (hereafter, we will use the term function
and program interchangeably). The function \( f \) may be
written in any language, but our running example will be
Matlab. We allow the function to be arbitrarily complex
inside, using any additional functions, recursion, lan-
guage constructs or external libraries it wishes. The only
constraint is that the function must be self-contained,
with no external side-effects which would impact the ex-
ecution of the function from one run to another.

The stochastic elements of \( f \) must come from a set of
known, fixed elementary random primitives, or ERPs. Complex distributions are constructed compositionally,
using ERPs as building blocks. In Matlab, ERPs may be
functions such as \( \text{rand} \) (sample uniformly from \([0,1]\)) or
\( \text{randn} \) (sample from a standard normal). Higher-order
random primitives, such as nonparametric distributions,
may also be defined, but must be fixed ahead of time.
Formally, let \( \mathcal{F} \) be the set of ERP types. We assume that
each type \( t \in \mathcal{F} \) is a parametric family of distributions
\( p_t(x|\theta_t) \), where \( \theta_t \) are the parameters of the distribution.

As \( f \) is executed, it encounters a series of ERPs. Algo-
rithm 1 shows an example of a simple \( f \) written in Matlab.
Here, there are three syntactic ERPs: \( \text{rand} \), \( \text{randn} \), and
gamma \( \text{rand} \). During execution, depending on the return
value of each call to \( \text{rand} \), different paths will be taken
through the program, and different ERPs will be en-
countered. We call this path an execution trace. A total of 2000
random choices will be made when executing this \( f \).

Let \( f_k[x_1,\ldots,x_k] \) be the \( k \)th ERP encountered while execu-
ting \( f \), and let \( x_k \) be the value it returns. Note that the
parameters passed to the \( k \)th ERP may change depend-
ing on previous \( x_i \)’s (indeed, its type may also change, as
well as the total number of ERPs). We denote by \( x \) all of
the random choices which are made by \( f \), meaning that \( f \)
defines the probability distribution \( p(x) \). In our example,
\( x \in \mathbb{R}^{2000} \). The probability of \( x \) is therefore the product of
the probability of all of the ERP choices made:

\[
p(x) = \prod_{k=1}^{K} p_{t_k}(x_k|\theta_{t_k},x_1,\ldots,x_{k-1})
\]

Algorithm 1 A simple Stochastic Matlab program repre-
senting a Gaussian-Gamma mixture model.

1: \hspace{1em} for \( i=1:1000 \)
2: \hspace{2em} if ( \( \text{rand} > 0.5 \) )
3: \hspace{3em} \( x(i) = \text{randn} \);
4: \hspace{2em} else
5: \hspace{3em} \( x(i) = \text{gamma}\text{rand} \);
6: \hspace{2em} end;
7: \hspace{1em} end;

again noting explicitly that types and parameters may de-
pend arbitrarily on previous random choices. To simplify
notation, we will omit the conditioning on the values of
previous ERPs, but again wish to emphasize that these
dependencies are critical and cannot be ignored. By \( f_k \),
it should therefore be understood that we mean \( f_k[x_1,\ldots,x_{k-1}] \),
and by \( p_{t_k}(x_k|\theta_{t_k}) \) we mean \( p_{t_k}(x_k|\theta_{t_k},x_1,\ldots,x_{k-1}) \).

Generative functions as described above are, of course,
easy to write. A much harder problem, and our goal in
this paper, is to reason about the posterior conditional
distribution \( p(x|c|x_c) \), where we define \( x_c \) to be a subset
of random choices which we condition on and \( x_{c} \) to be
the remaining random choices. For example, we may
condition our example program on the values of
the \( x(i) \)’s, and reason about the sequence of \( \text{rand} \)’s most
likely to generate the \( x(i) \)’s. Rejection sampling is one
possible inference method: we may run \( f \) forward until
\( x_c \) is generated, somehow recording \( x_{c} \). Of course, such
an approach is intractable for all but the simplest models.

We propose a different approach, based on the follow-
ing intuition. Consider repeatedly running an uncondi-
tioned \( f \). Every time an \( f_k \) is encountered, the program
samples an \( x_k \) and continues execution. Importantly, if
two runs of \( f \) sample exactly the same values, their traces
will be the same. This suggests the following procedure
for controlling the execution of \( f \):

1. Give each \( f_k \) in every possible execution trace a
   “name.” This name does not have to be unique, and
can depend on previous \( x_k \)’s or other information
about program state.

2. Rewrite the source code of \( f \) to generate \( f' \), replac-
ing random functions \( f_k \) with deterministic functions
\( f'_k \). When encountered in the execution trace,
these functions \( f'_k \) deterministically use their name
to look up a current value \( x_k \) in a database and re-
turn it (and if no value exists, they sample \( x_k \sim
p_{t_k}(\cdot|\theta_{t_k}) \), store it in the database, and then return
it). Behind the scenes, these functions also accumu-
late the likelihood of each random choice.

Thus, the execution trace of \( f' \) can be controlled by ma-
nipulating the values in the database. This makes MCMC
Algorithm 2 An MCMC trace sampler.

1: Initialize: $[\mathcal{D}, \Pi_i, \ldots] = \text{trace}_\text{update}(\emptyset)$
2: Repeat forever:
3: Select a random $f_k$ via its name $n$
4: Look up its current value $(t, x, l, \theta_{db}) = \mathcal{D}(n)$.
5: Propose a new value $x' \sim \mathcal{K}_i(x|\theta_{db})$
6: Compute $F = \log \mathcal{K}_i(x'|x, \theta_{db})$
7: Compute $R = \log \mathcal{K}_i(x|x', \theta_{db})$
8: Compute $l' = \log p_t(x'|\theta_{db})$
9: Let $\mathcal{D}' = \mathcal{D}$
10: Set $\mathcal{D}'(n) = (t, x', l', \theta_{db})$
11: $[\mathcal{D}', \Pi'_i, \Pi'_\text{fresh}, \Pi'_\text{stale}] = \text{trace}_\text{update}(\mathcal{D}')$
12: $\alpha = 1 + R + F + \log |\mathcal{D}| - \log |\mathcal{D}'| + \Pi'_\text{stale} - \Pi'_\text{fresh}$
13: if $(\log(\text{rand}) < \alpha)$ then
14: $\mathcal{D} = \mathcal{D}'$
15: $\Pi = \Pi'_i$
16: else
17: $\Pi = \Pi'_i$
18: end

Algorithm 3 Function trace_update ( $\mathcal{D}$ )

1: Initialize $\Pi_i, \Pi_\text{fresh}$, and $\Pi_\text{stale}$ to 0
2: Execute $f$:
3: For all random choices $k$:
4: Run computation until choice $k$,
5: determining $n, t, c, \theta_k$ for $k$.
6: Look up $(t, x, l, \theta_{db}) = \mathcal{D}(n)$
7: if a value is found in the database and $t = t_c$
8: if parameters match (ie, $\theta_c = \theta_{db}$)
9: $l = l + 1$
10: else
11: Compute $l = \log p_t(x|\theta_c)$
12: Store $\mathcal{D}(n) = (t_c, x, l, \theta_c)$
13: $l = l + 1$
14: else
15: Sample new randomness $x \sim p_i(x|\theta_c)$
16: Compute $l = \log p_t(x|\theta_c)$
17: Store $\mathcal{D}(n) = (t_c, x, l, \theta_c)$
18: $l = l + 1$
19: $\Pi'_\text{fresh} = \Pi'_\text{fresh} + 1$
20: Set return value of $f_k$ to $x$
21: For all random choices $k$ in $\mathcal{D}$
22: if $k$ has not been active in this update:
23: Look up $(t, x, l, \theta_{db}) = \mathcal{D}(k)$
24: $\Pi'_\text{stale} = \Pi'_\text{stale} + \log p_t(x|\theta_{db})$
25: Remove $k$ from $\mathcal{D}$
26: return $[\mathcal{D}, \Pi_i, \Pi_\text{fresh}, \Pi_\text{stale}]$

We now address the details. Section 2.1 discusses the overall MCMC algorithm, Section 2.2 discusses how random variables are named, and Section 2.3 discusses conditional inference.

2.1 MCMC IN TRACE SPACE

We now describe our MCMC algorithm. Here, we assume that some naming scheme has been defined, and that a mechanism for computing those names has been implemented; we will discuss how to do this in the next section.

Let a database $\mathcal{D}$ be defined as a mapping $\mathbb{N} \rightarrow \mathcal{F} \times \mathcal{X} \times L \times \theta$, where $\mathcal{N}$ is the name of a random choice, $\mathcal{F}$ is its ERP type, $\mathcal{X}$ is its value, $\theta$ are ERP parameters, and $L$ is this random value's likelihood. We allow missing entries.

We can control the execution of $f'$ by controlling the values in $\mathcal{D}$. When $f'$ encounters an $f'_k$, it computes its name $n \in \mathcal{N}$, its parameters $\theta_c$ (where ‘c’ is a mnemonic for “current”), and its current type $t_c \in \mathcal{F}$. It then looks up $(t, x, l, \theta_{db}) = \mathcal{D}(n)$. If a value is found and the types match, we set the return value of $f_k$ to be $x$. Otherwise, the value is sampled from the appropriate ERP $x \sim p_t(x|\theta_c)$, its likelihood is computed, and the corresponding entry in the database is updated. We accumulate the log-probability of new randomness, $\Pi'_\text{fresh}$, and the log-probability of randomness that is no longer used, $\Pi'_\text{stale}$. This is formalized in the trace_update procedure, defined in Algorithm 3.

We use trace_update to define our overall MCMC algorithm, shown in Algorithm 2. Given a current trace $x$ and score $p(x)$, we proceed by reconsidering one random choice $x_k$. We equip each ERP type with a proposal kernel $\mathcal{K}_i(x'|x, \theta)$, which we use to generate proposals to $x_k$. After a proposal, we call trace_update to generate a new trace $x'$, and compute its likelihood $p(x')$ and the probabilities of new and stale randomness, $p_t$ and $p_{\text{stale}}$. This gives us an overall score, which is used as an MH accept ratio

$$\alpha = \min \left\{ 1, \frac{p(x') \mathcal{K}_i(x'|x, \theta) |D| p_{\text{stale}}}{p(x) \mathcal{K}_i(x'|x, \theta) |D'| p_{\text{fresh}}} \right\}$$

There are a few subtleties to the implementation. Even if $x$ for an $f'_k$ is found in $\mathcal{D}$, we must compare the pa-
Algorithm 4 An illustration how the number and type of random choices can change.

1: m = poissrnd();
2: for i=1:m
3:   X(i) = gammaRand();
4: end;
5: for i=m+1:2*m
6:   X(i) = randn;
7: end;

rameters $\theta_c$ and $\theta_{db}$ to make sure they are equal; if they are not, we may still be able to re-use $x$, but must re-score it under the new parameters $\theta_c$. We also remove unused (“stale”) random choices from the database to ensure that subsequent new randomness is sampled from the prior, and allows us to avoid reversible jump corrections. Finally, note that many of the probabilities in $\alpha$ cancel when proposals are made from the local prior.

### 2.2 NAMING RANDOM VARIABLES

What properties do we desire in a naming scheme? We build around a simple idea: when we propose a new $x'_k$ and update the trace, we wish to reuse as many $x_k$’s as possible. This will give us a new execution trace “close” to the original, which is important to ensure a high acceptance rate in our MCMC sampler (recall that if an $f'_k$ cannot reuse a value from $D$, one is sampled from the prior, which is unlikely to be good).

How might different naming schemes affect reuse of randomness? Algorithm 4 shows a simple program $f$ which we will use to illustrate different approaches. As $f$ executes, it first encounters a Poisson, and then a sequence of gamma and Gaussian variables. One might be tempted to name random variables sequentially, according to the order in which they are encountered during execution—that is, use $k$ itself as the name. Figure 1 (top) illustrates why this is a bad idea. Two traces are shown schematically, illustrating the $f'_k$’s encountered. Suppose that in trace #1, the value of $n$ is 3. A proposal is made, changing $n$ to 2, yielding trace #2. If variables are named sequentially, then $x_1$ and $x_2$ can be reused without problem, but when $f'_{1,3}$ looks itself up in $D$, it discovers that its type has changed, and must sample $x_3$ from the prior. The rest of the sequence is now “misaligned,” and must probably be discarded. More sophisticated versions of this problem complicate this naming scheme to the point of unworkability, especially with complex models.

What is an alternative? Examining Alg. 4, we see that would like to name variables according to the way they are used at a more semantic level. For example, we might want to name a variable “the third gamma variable in the loop on lines 2-4.”

Our key technical contribution formalizes this intuition. We name random choices according to their structural position in the execution trace, which we define in a way roughly analogous to a stack address: a variable’s name is defined as list of the functions, their line numbers, and their loop indices that precede it in the call stack. Thus, in Alg. 4, variables are named “1” (the initial Poisson), “3-1,” “3-2,” ... (the gamma variables sampled on line 3), and “6-1,” “6-2,” ... (the Gaussian variables sampled on line 6). Figure 1(bottom) illustrates this on Alg. 4: we see that the maximum number of random variables are reused.

We now discuss naming schemes for imperative and functional languages.

**Imperative.** For an imperative language, such as Matlab, we define the structural position with an abstract stack trace that is augmented with a combination of function identifiers, line numbers, and loop iteration numbers. The formal specification is inductive, as shown in Figure 2. In an interpreted imperative language, such as Python, Matlab or Ruby, this naming information could potentially be acquired via dynamic stack examination, assuming suitable libraries exist. Alternatively, this information can be provided by augmenting the source code for $f$, as we shortly discuss.

**Functional.** For a functional language, such as Lisp, we do not need to worry about loops or line numbers: everything is a function, so we may define the structural position with a stack trace that is augmented with function identifiers. The specification is shown in Figure 3.

There is no guarantee that this naming scheme is optimal in any sense, but it does capture our desiderata, is simple and fast, and incurs minimal overhead.
The imperative naming specification. The name of an \( f_k \) is the state of the three stacks when \( f_k \) is encountered.

2.3 CONDITIONAL INference

In principle, a program can be conditioned on any expression that evaluates to a Boolean value, not just on a fixed value assignment for a subset of the random variables of the program. The meaning of such a conditioned program is defined by rejection sampling: Sample an execution trace from the unconditioned program. If the value of the conditioning expression is true, return this trace. Otherwise, retry.

In practice, a MCMC implementation that allows general conditions requires that for any valid condition, we can find an initial assignment of random variables to values that satisfies this condition. If conditioning is not restricted to random variables (Stochastic Matlab), then this is a search problem that needs to be solved. Possible approaches include constraint propagation (MIT-Church), annealing (Bher), and rejection sampling (Bher). We leave the analysis of this problem to future research.

The naming specification for Bher is given by a syntactic transform \( \mathcal{A}^{top} \).

\begin{align*}
\mathcal{A}^{top}[E] &= ((\lambda (addr) \mathcal{A}[E]) \ 'top') \\
\mathcal{A}[(\lambda (I^n_{i=1}) E_{body})] &= (\lambda (addr . I^n_{i=1}) \mathcal{A}[E_{body}]) \\
\mathcal{A}[(\text{if } E_{if} E_{else})] &= ((\lambda \mathcal{A}[E_{if}] \mathcal{A}[E_{else}]) \\
\mathcal{A}[(\text{define } I E)] &= (\text{define } I \mathcal{A}[E]) \\
\mathcal{A}[\{quote E\}] &= \{quote E\} \\
\mathcal{A}[\{E_{op} E^n_{i=1}\}] &= (\mathcal{A}[E_{op}] \ \text{cons} \ S \ \text{addr} \ \mathcal{A}[E_{i=1}]) \ \text{with} \ S \ \text{being} \ \text{a globally unique symbol.} \\
\mathcal{A}[E] &= E, \ \text{otherwise.}
\end{align*}

Figure 3: The imperative naming specification. The name of an \( f_k \) is the state of the three stacks when \( f_k \) is encountered.

\[ \mathcal{L}[\{\lambda (I^n_{i=1}) E_{body}\}] = (\lambda (I^n_{i=1}) \mathcal{L}[E_{body}]) \]
\[ \mathcal{L}[\{\text{letrec } ((I_{i=1}) E_{i=1}) E\}] = (\text{letrec } ((I_{i=1}) \mathcal{L}[E_{i=1}]) \mathcal{L}[E]) \]
\[ \mathcal{L}[(\text{begin } E_{i=1})] = (\text{begin} \ \text{force} \ \mathcal{L}[E_{i=1}]) \]
\[ \mathcal{L}[(\text{mem } E)] = (\text{mem} \ \mathcal{L}[E]) \]
\[ \mathcal{L}[\{quote E\}] = \{quote E\} \]
\[ \mathcal{L}[\{\text{if } E_{if} E_{else}\}] = (\text{if} \ \text{force} \ \mathcal{L}[E_{if}] \ \mathcal{L}[E_{else}]) \]
\[ \mathcal{L}[(\text{Eop } E^n_{i=1})] = (\text{force} \ \mathcal{L}[E_{op}]) \]
\[ \mathcal{L}[E] = E, \ \text{otherwise.} \]
\[ \mathcal{D}[\{\lambda (I E)\}] = \mathcal{L}[\{\lambda (I E)\}] \]
\[ \mathcal{D}[\{\text{mem} \ (\lambda (I E))\}] = \mathcal{L}[\{\text{mem} \ (\lambda (I E))\}] \]
\[ \mathcal{D}[E] = \text{list delayed} \ (\text{mem} \ (\text{cons} \ S \ \text{addr})) \ (\mathcal{L}[E]), \ \text{otherwise.} \]

\begin{align*}
\text{(define (force address val)} \\
&\quad \text{(if (and (pair? val) (eq? (car val) 'delayed))} \\
&\quad \quad \text{(force address ((cadr val) address)))} \\
&\quad \text{(if (force address delay (eq? 'delayed))} \\
&\quad \quad \text{(force address ((cadr val) address)))}) \\
&\quad \text{val})
\end{align*}

Figure 4: Laziness transform \( \mathcal{L} \) and definition of the function force referenced in the transform.

3 IMPLEMENTATION AND EXPERIMENTS

We now present two implementations of our technique. Bher is a compiled implementation of Church, and Stochastic Matlab is a new, open-source language. Both illustrate the virtues of our approach: small code bases, high speed, and language flexibility.

3.1 EXAMPLE: BHER

Bher is an implementation of the Church probabilistic programming language that compiles Church code to native Scheme code. Like MIT-Church, the original implementation of the Church language, Bher is written in R6RS Scheme. However, MIT-Church is an interpreter—not based on the lightweight implementation principle—and thus provides a useful point of comparison.

In the following, we first discuss the addressing transform that names random choices in Bher, then compare performance between Bher and MIT-Church on Hidden Markov Models and Latent Dirichlet Allocation.
is very different from the distribution defined by \((\text{eq?} \ (\text{flip}) \ (\text{flip}))\).

The distribution defined by \((\text{mem} \ \text{flip})\)

In stochastic programs, we can distinguish

In deterministic programs, memoization is an optimization technique [4]. The distribution defined by \((\text{eq?} \ (\text{flip}) \ (\text{flip}))\) is very different from the distribution defined by \((\text{let} \ [(\text{mf} \ (\text{mem} \ \text{flip})) \ (\text{eq?} \ (\text{mf}) \ (\text{mf}))])\). The former is true with probability .5, false with probability .5; the latter is true with probability 1.

When addresses are available, semantic memoization is a simple syntactic transform. In a program that has been transformed to provide addresses, random variables get their values by looking up the value that is stored in the global database at their address. Thus, to enforce that two random choices always have the same value, we make sure that the address they receive is the same. To achieve this, the mem case of the addressing transform builds a function that captures within its closure the address of the mem creation and uses this address (extended by any function arguments) instead of the address provided to the function when it is applied.

The Church program shown in Algorithm 5 and its transformed version (Algorithm 6) illustrate the address computation that is induced by the syntactic transform. The program defines a simple geometric distribution with a potentially unbounded number of random choices, each of which gets a unique address. When we propose to change the value of the flip that controls the recursion depth, we want to reuse as much of the existing trace as possible. An example of such a trace is shown in Figure 5. Note that apply nodes (shown in green) are the only place where addresses change and that nodes that refer to the same syntactic place (e.g. the two blue \((\text{flip} \ p)\) nodes) have different addresses when they occur in different places in the trace. A change to the return value of such a flip does not affect the addresses of the choices “above” it. Thus, any randomness stored for these choices persists across the change.

A major virtue of source-to-source transformations is simplicity and compositional-ity. To illustrate this point, we now sketch a complementary transformation, eager style to lazy style, that can be used to avoid unnecessary computation. As shown in Figure 4, we delay expressions that must not necessarily get evaluated—for example, arguments to compound functions—and force expressions that are always evaluated—operators, for example. To delay an expression E, we wrap it in \((\text{mem} \ (\lambda (\text{args}) \ \text{apply} \ \text{prim} \ \text{args}))\). Thus, we ensure that it always returns the same result when evaluated multiple times. To force a value, we check whether it is delayed and if it is, we simply apply the memoized function stored in the value and recurse.

This difference in simplicity manifests itself in the code base of the two projects. While the MIT-Church imple-
3.1.2 Experiments

We have tested inference performance in MIT-Church and Bher on Hidden Markov Models and Latent Dirichlet Allocation (Figure 6). In both cases, we conditioned on an observation sampled from the model. For each HMM, we generated a sentence of length 15. For LDA, we used 10 topics, 10 words per document, and 30 possible words, varying the number of observed documents. All experiments use the optimizing compiler Ikarus [3].

On the HMM model, Bher outperforms the MIT-Church interpreter. On the LDA model, Bher shows better performance at smaller problem sizes whereas MIT-Church scales better to larger instances. This difference in scaling is due to (1) reuse of cached computation in \texttt{mem} and (2) computational short-circuiting in MIT-Church. In the process of updating a trace after a proposal has been made, MIT-Church can detect that the information flowing into a function application has not changed; in this case, it does not recompute the return value. Since \texttt{mem} is transformed away by the compiler, perhaps there is a source transformation that results in similar reuse of computation. This is an area of future research.

3.2 EXAMPLE: STOCHASTIC Matlab

We now illustrate our technique to create a new, imperative probabilistic programming language called “Stochastic Matlab,” which can take advantage of Matlab’s rich native libraries and toolboxes, and its parallelization, profiling and debugging support.

To rewrite \( f \), we used the imperative ERP naming specification as outlined in Fig. 2. We implemented the rewriter using a parser developed with code from the open-source Octave project [2]. ERP names were rewritten to deterministic functions using a MEX-based database. Remaining code was implemented in pure Matlab.

We now present a case study that relies on a deterministic GPU-based MEX function. The point is twofold: first, to show how easy it is to mix arbitrary deterministic elements with stochastic ones—reusing complex external libraries—and second, to show how easy it can be to create complex models with a few simple commands.

Our example is a small (but real) application of inverse geological modeling. Layers of rock and sediment are deposited over millions of years to create a volumetric cube of rock with porosity that varies according to the geometries of the layers. Given this forward model, we wish to condition on a given volume, and recover a description of the individual layers that built up the volume.

Figure 7 shows the model, which generates a rock volume by first sampling parametric descriptions for each layer (the details are not shown; layers are described as a weighted set of basis functions), and then “rendering” them into a porosity volume (with Gaussian noise added at each voxel). The render_layers function encapsu-
A layer model of rock porosity

```matlab
function dp_render_rock( data )
    num_layers = poissrnd();
    dp_sample_layer = dpmem( @sample_layer, 1.0 );
    for i=1:num_layers
        layer(i,:) = dp_sample_layer();
    end;
    rock_volume = render_layers( layers );
    data = rock_volume + randn( size(rock_volume) );
    return;
```

![True porosity and inferred porosity](image)

Figure 7: Results on modeling rock porosity.

lates the MEX/GPU-based renderer. Given this model, we perform inference using tempered MCMC, conditioning on the data shown in Fig. 7. A sample from the posterior shows a reconstructed volume (also shown in Fig. 7) that closely matches the true volume—the program has successfully inferred a parametric form for each layer.

The program calls a function named `dpmem`, a stochastic memoizer similar to that found in Church. `dpmem` accepts a function representing a base measure and a concentration parameter, and returns a new function which samples according to a Dirichlet process (DP). We use this to create a DP in “layer space,” where parameters of layers can be shared. Fig. 7 (lower left) shows a comparison of inference with and without the `dpmem`: shared layers enable the inference engine to find higher-likelihood models faster. Hierarchical DPs are equally easy: we could (for example) let \( f = \text{dpmem}(\text{randn}, 1.0) \), and \( g = \text{dpmem}(\text{rand}, 1.0) \) to create an HDP with a Gaussian base measure. We consider such flexibility in defining models—combined with the rich libraries of Matlab—to be the major virtue of our approach.

4 RELATED WORK

Our MCMC algorithm is a new implementation of the Church MCMC algorithm [4], and is similar to the algorithm used in BLOG [7]. However, we provide a simpler and more uniform approach to constructing such algorithms for new languages, with small, clean code bases.

The closest related work is that of Kiselyov and Shan [6], who share the goal of transforming standard languages into probabilistic versions with little interpretive overhead. They use delimited continuations to control random choices without explicit choice naming. However, their technique is only applicable (prima facie) to enumeration or importance sampling, not MCMC.

Other related approaches include BUGS [16], HBC [1], and Infer.NET [8]. BUGS is not transformational (it simply loads a data structure into memory from a config file), but the implementations of both Infer.NET and HBC are comparable to our approach. Infer.NET uses a complicated transformation to compile csoft to C#. HBC compiles from a config file to C, but the specification language used by HBC is not a full-fledged language. We consider our approach simpler and more general.

With respect to modeling flexibility, our approach provides a simple dynamic knowledge-based model construction, which allows unbounded models, including complex recursions, to be handled efficiently. Models such as the naive implementation of the geometric distribution in Bher (Alg. 5), the gamma/randn example in Stochastic Matlab (Alg. 4), and nonparametric distributions such as the Dirichlet Process (Alg. 7) cannot be specified in these other languages.

A performance comparison between these languages is important, but beyond the scope of this paper. A careful comparison must account for issues such as mixing rates, initialization values, differences in parameters of proposal kernels, etc. and so we have explicitly limited this paper to an explanation of the technique, with comparisons reserved for a longer future paper.

5 CONCLUSION

We have described a lightweight implementation technique for probabilistic programming languages. The method is simple and fast, and the resulting languages permit a great deal of flexibility in the specification of distributions by mixing stochastic and deterministic elements with arbitrary language features (such as objects, inheritance, operator overloading, closures, recursion, libraries, etc.). Our example implementations have compact code bases and reasonable inference speed.

The main directions for improvement are better mixing and faster inference. Because source-to-source transformations are often compositional, more transformations could be applied to enhance performance. These could reduce redundant computation between traces, generate compound proposals, or implement constraint propagation for initializing conditioners. Future work will investigate these issues, as well as the possibility of new languages, such as stochastic Python.
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References


